

Gainesville College

Round 3: $x + y = 2 \Rightarrow (x + y)^2 = (2)^2$. So,

$$x^2 + y^2 + 2xy + 2x + 2y = 4 \Rightarrow x^2 + y^2 + 2(xy + x + y) = 4.$$

Since $x + y = 1$, we have

$$x^2 + y^2 + 2 = 4 \Rightarrow x^2 + y^2 = 2.$$

Round 4: —

Round 6: The total number of crossing points is $10 \times 7 = 70$. The total number of ordered pairs of points is $70 \times 69 = 4830$. The number of ordered horizontal pairs is $7 \times 6 \times 10 = 420$. The number of ordered vertical pairs is $10 \times 9 \times 7 = 630$. The number of ordered non-vertical, non-horizontal pairs is $4830 - 420 - 630 = 3780$.

Round 10: Let $\theta = \arcsin\left(\frac{4}{3}\right)$. Then the triangle at the right could be drawn.

By the Pythagorean Theorem, the length of the hypotenuse is 5, and

$$\sin \theta = \frac{4}{5}. \text{ Therefore,}$$

$$\left(\arcsin\left(\arcsin\left(\arcsin\left(\arcsin\left(\arcsin\left(\frac{4}{3} \right) \right) \right) \right) \right) \right) = \arcsin\left(\arcsin\left(\arcsin\left(\frac{4}{5} \right) \right) \right)$$

$$\text{Now, let } \phi = \arcsin\left(\frac{4}{5}\right)$$