Gainesville State College

Fourteenth Annual Mathematics Tournament

April 12, 2008

Solutions for the Afternoon Team Competition

Round 1

Let x be the number of rabbits, y be the number of kittens, and z the number of chickens. We have

$$(1)\ \ x + y + z = 100$$

$$(2)\ \ 2x + y + 0.1z = 100$$

The third condition gives us $z = \frac{2}{3}(x + y)$ or $3z = 2x + 2y$.

Multiply equation (1) by 2 to obtain $2x + 2y + 2z = 200 \Rightarrow 3z + 2z = 200 \Rightarrow z = 40$.

Substituting back and reducing, we have

$$x \quad y \quad 60$$

Round 3

To get a zero at the end of a number, you need to multiply a 2 and a 5 together. There are fewer factors of 5 in numbers between 1 and 100 than there are factors of 2. So the number of factors of 5 contained in 100! determines the number of zeros at the end.

100! = (the other integers without factors of 5)$\left(100 \cdot 95 \cdot 90 \cdot \ldots \cdot 5\right)$.

The tables show all the integers with factors of 5 that are in 100!

Round 4

$$\angle A + \angle 1 + \angle D = 180^\circ$$

$$\angle B + \angle 2 + \angle E = 180^\circ$$

$$\angle C + \angle 3 + \angle A = 180^\circ$$

$$\angle D + \angle 4 + \angle B = 180^\circ$$

$$\angle E + \angle 5 + \angle C = 180^\circ$$

$$2\left(\qquad \qquad \qquad \right)$$

Round 5

The discriminant is $b^2 - 4c$. If $b^2 - 4c < 0$, then $b^2 < 4c$ and the quadratic equation has no real solutions. Consider the following:

1. When $b = 1$, $b^2 = 1 \Rightarrow 1 < 4c \Rightarrow c > \frac{1}{4}$. So $c = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.
   Thus, 10 such equations.

2. When $b = 2$, $b^2 = 4 \Rightarrow 4 < 4c \Rightarrow c > 1$. So $c = 2, 3, 4, 5, 6, 7, 8, 9, 10$.
   Thus, 9 such equations.

3. When $b = 3$, $b^2 = 9 \Rightarrow 9 < 4c \Rightarrow c > \frac{9}{4}$. So $c = 3, 4, 5, 6, 7, 8, 9, 10$.
   Thus, 8 such equations.

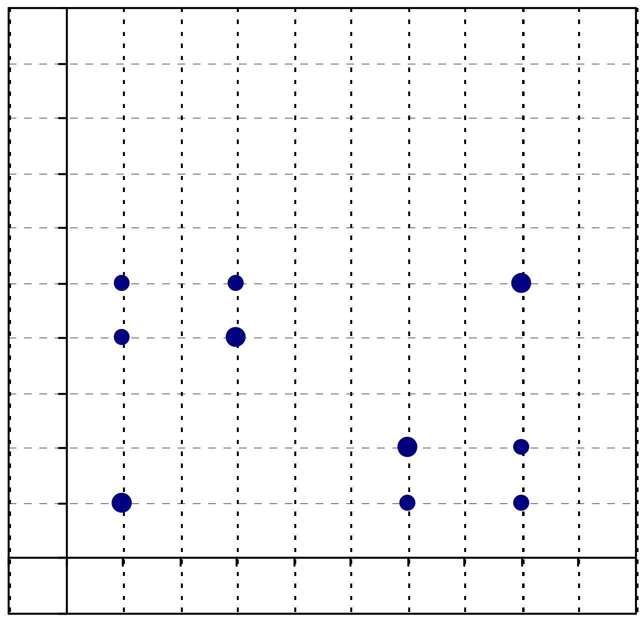
4. When $b = 4$, $b^2 = 16 \Rightarrow 16 < 4c \Rightarrow c > 4$. So $c = 5, 6, 7, 8, 9, 10$.
   Thus, 6 such equations.

5. When $b = 5$, $b^2 = 25 \Rightarrow 25 < 4c \Rightarrow c > \frac{25}{4}$. So $c = 7, 8, 9, 10$.
   Thus, 4 such equations.

6. When $b = 6$, $b^2 = 36 \Rightarrow 36 < 4c \Rightarrow c > 9$. So $c = 10$.
   Thus, 1 such equation.

7. When $b = 7$, $b^2 = 49 \Rightarrow 49 < 4c \Rightarrow c > \frac{49}{4} > 10$. So there are no more considerations.

Hence, there are a total of $10 + 9 + 8 + 6 + 4 + 1 = 38$ such equations.



Round 7

$$\frac{1\cdot 2\cdot 4+2\cdot 4\cdot 8+3\cdot 6\cdot 12+}{1\ \ 3\ \ 9\quad 2\ \ 6\ \ 18\quad 3\ \ 9\ \ 27}^{\frac{1}{3}} = \frac{1\cdot 2\cdot 4\left(1^3+2^3+3^3+\quad\right)}{1\ \ 3\ \ 9\left(1^3\quad 2^3\quad 3^3\quad\right)}^{\frac{1}{3}} = \frac{8}{27}^{\frac{1}{3}} = \frac{2}{3}$$