3. A designer wants to introduce a new line of bookcases. He wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is $C(x)$. Assume that $C(x)$ is a differentiable function. Which of the following must he do to find the minimum average cost, $c(x) = \frac{C(x)}{x}$?

(I) Find the points where $c'(x) = 0$ and evaluate $c(x)$ there.

(II) Compute $c''(x)$ to check which of the critical points in (I) are local maxima.

(III) Check the values of c at the endpoints of its domain.

a) I only
b) I and II only
c) I, II and III
d) I and III only
e) none of the above

4. If $|x|$ is very large, then the graph of $f(x) = \frac{2x^3 - 3x^2 + x + 5}{x^2 - x + 1}$

6. A particle is moving in the first quadrant downward on the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$. It leaves the hyperbolic path at the point $(5, 6)$ and continues along a straight line. At what point does the particle cross the x-axis?

a) $\frac{16}{}, 0$

9. Calculate the derivative of the function $f(x) = x^{\left(e^x\right)}$.

a) $e^x x^{\left(e^x - 1\right)}$

b) $x^{\left(e^x\right)}\left[e^x \ln x + \frac{e^x}{x}\right]$

c) $x^{\left(e^x - 1\right)} e^x \left[\frac{1}{x} + \ln x\right]$

d) $e^x x^{\left(e^x - 1\right)} \ln x$

e) none of the above

12. A hemispherical bowl of radius a contains water to depth h. Find the volume of the water in the bowl.

14. If f and g are both differentiable and $h = f \circ g$, $h'(2)$ equals

a) $f'(2) \circ g'(2)$
b) $f'(2)\, g'(2)$
c) $f'(g(x))\, g'(2)$
d) $f'(g(2))\, g'(2)$
e) none of the above

15. Let $f(x) = x|x|$. Find $f'(0)$.

a) 0
b) 1
c) -1
d) does not exist
e) none of the above

16. Evaluate: $\lim_{x\to 0} \frac{1-e^{-x}}{\sin x}$

a) ∞
b) 1
c) $\sqrt{3}$
d)

18. Evaluate: $\int_{0}^{-1} \frac{1}{\sqrt{x+1+\sqrt{x+1}}}\,dx$

a) $-2\sqrt{2}+\ln\left(3+2\sqrt{2}\right)$

b) $\frac{1}{-2\sqrt{2}}+\ln\left(3+2\sqrt{2}\right)$

c) $-2\sqrt{2}+\ln\left(2+3\sqrt{3}\right)$

d) $\frac{1}{-2\sqrt{2}}+\ln\left(2+3\sqrt{3}\right)$

e) none of the above

19. Find the length of the curve $x=\frac{y^4}{4}+\frac{1}{8y^2}$ from $y=1$ to $y=2$.

a) $\frac{}{}$

21. Evaluate: $\int_0^1 \sqrt{\frac{1+x}{1-x}}\, dx$

a) $\pi - 1$

b) $\frac{\pi}{2} - 1$

c) $\frac{\pi}{2} + 1$

d) $\frac{\pi}{4} + 1$

e) none of the above

22. If $f(x) = \ln|Cx|$, for $C \neq 0$, then $f'(x) =$

a) $\frac{1}{|x|}$

b) $\frac{1}{|Cx|}$

c) $\frac{1}{x}$

d) $\frac{1}{Cx}$

e) none of the above

23. If a trigonometric substitution in terms of a secant function in the variable θ is used to solve $\int_{\frac{5}{2}}^{\frac{5}{\sqrt{3}}} \sqrt{4x^2 - 25}\, dx$

24. What is the area of the largest rectangle that can be inscribed in the region bounded by $y = 3 - x^2$ and the x-axis?

a) 4
b) 6
c) $\frac{3\pi}{2}$
d) $\sqrt{5}$

26. A ball is thrown straight up from the ground. How high will it go? Assume that g is the absolute value of the gravitational acceleration and v_0 is the initial velocity.

a) $g{v_0}^2$

b) $\frac{1}{2}g^2 + gv_0$

c) $\frac{1}{2}v_0 + \frac{1}{2}{v_0}^2 g$

d) $\frac{1}{2}{v_0}^2 g^{-1}$

e) none of the above

27. Given $F(x) = \int_0^{x^2} e^{5t-t^2}\,dt$, find $F'(2)$.

a) $4e^4$

b) $-3e^4$

c) $-3e^4 - 5$

d) $\frac{1}{5}e^4 - e^{10}$

e) none of the above

28. Two electrons repel each other with a force inve

29. If $f(x) = \sqrt{x} - x + 9$, for $x \geq \frac{1}{2}$, and $g = f^{-1}$, then $g'(9)$ is

a) -2

b) $-\frac{5}{6}$

c) $-\frac{6}{5}$

d) -1

e) none of the above

30. Integrate: $\int x \ln(x^2) dx$.

a) $\frac{x^2 \ln(x^2)}{2} - \frac{x^3}{3} + C$

b) $\frac{x^2 \ln(x^2)}{2} - \frac{x^2}{2} + C$

c) $\frac{x^2 \ln(x^2)}{2} + \frac{x^2}{2} + C$

d) $\frac{x^2 \ln x}{2} - \frac{x}{2} + C$

e) none of the above

31. Evaluate: $\lim_{x \to \sqrt[3]{2}} \left(\frac{x^2}{2} - \frac{1}{x} \right)$

a) $+\infty$

b) $\frac{2}{3} \cdot \frac{1}{\sqrt[3]{2}}$

c) 0

d) $\frac{3}{2} \cdot \sqrt[3]{2} - \frac{1}{\sqrt[3]{2}}$

e) none of the above

32. If the product function $h(x) = f(x) \cdot g(x)$ is continuous at $x = 0$, then the following must be true about the functions f and g. (Choose just one answer.)

a) Both functions must be continuous at $x = 0$.
b) One of them must be continuous at $x = 0$, but not necessarily the other.
c) Both must be discontinuous at $x = 0$.
d) They may be continuous or not at $x = 0$, all options are possible.
e) none of the above

33. One way to compute $\frac{1}{2}$ the area of the unit circle is to integrate

34. Evaluate: $\int_{-1}^{1} \frac{dx}{x^2 - 6x + 9}$

37. How many zeros does the function $f(x) = x^4 + 3x + 1$ have in the interval [-2, -1]?

a) no zeros
b) exactly one zero
c) exactly two zeros
d) exactly three zeros
e) none of the above

38. Given that $f(n) = \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + ... + \frac{1}{\sqrt{n}\sqrt{n+n}}$. Find $\lim_{n \to \infty} f(n)$.

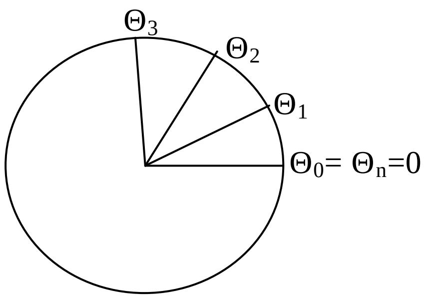
a) $1 + \frac{1}{e}$

b) $\sqrt{2}$ [redacted]

c) $1-$ [redacted] e

d) 2 [redacted] $\sqrt{2}$ 2

40. We cut a circular disk of radius r into n circular sectors as shown in the figure, by marking the angles θ_i at which we make the cuts ($\theta_0 = \theta_n$ can be considered to be the angle 0). A circular sector between two angles θ_i and θ_{i+1} has an area $\frac{1}{2}r^2\Delta\theta_i$, where $\Delta\theta_i = \theta_{i+1} - \theta_i$.



We let $A_n = \sum_{i=0}^{n-1}\frac{1}{2}r^2\Delta\theta_i$. Then the area of the disk, A, is given by:

a) A_n, independent of how many sectors we cut the disk into

b) $\lim_{n\to\infty} A_n$

c) $\int_0^{2\pi}\frac{1}{2}r^2 d\theta$

d) all of the above

e) none of the above