SixteenthAnnual Gainesville State College
Mathematics Tournament

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

3. The area of a circular oil spill is increasing at the rate of 40 square kilometers per hour. At what rate is the radius of the spill increasing, when the radius is 4 kilometers?

a) $\frac{\phi}{5}\ {}^{km}\!/\!_{hr}$

b) $\frac{5}{\phi}\ {}^{km}\!/\!_{hr}$

c) $10\phi\ {}^{km}\!/\!_{hr}$

d) $\frac{10}{\phi}\ {}^{km}\!/\!_{hr}$

e) none of the above

4. Find $\int \arcsin(3x)\,dx$.

a) $\arcsin(3x) 2 C$

b) $\frac{3}{\sqrt{14\ 9x^2}} 2 C$

c) $x\arcsin(3x) 2 \frac{3}{\sqrt{14\ 9x^2}} 2 C$

d) $x\arcsin(3x) 2 \frac{\sqrt{14\ 9x^2}}{3} 2 C$

e) none of the above

5. A wire of length L is cut into two pieces. One piece, of length x, is bent to form a circle, and the other to form a square. Find x so that the sum of the areas of the circle and the square is a minimum.

a) $x \mid \frac{L}{2\,2\,\phi}$

b) $x \mid \frac{\phi L}{4\,2\,\phi}$

c) $x \mid \frac{\phi^2 L}{(2\,2\,\phi)^2}$

d) $x \mid \frac{\phi L}{2\,2\,\phi}$

e) none of the above

6. If $c \} 0$ and $f(x) \mid e^x \, 4\, cx$, then the minimum value of f is

a) $f(c)$

b) $f\,/e^c\,0$

c) no minimum exists

d) $f\,/\ln c\,0$

e) none of the above

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7. Find the maximal area of a rectangle inside a right isosceles triangle with legs of length 1 as indicated in the picture below:

a) $\frac{1}{2}$

b) $\frac{\sqrt{2}}{2}$

c) $\frac{1}{3}$

d) $\frac{1}{4}$

e) none of the above

8. Find the limit: $\lim_{x \to 1} \left(\frac{1}{x}\right)^{1/\ln(x)}$.

a) e^{-1}
b) 1
c) e
d) 0

e) none of the above

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14. Suppose f is an even function differentiable for all real numbers. What can be said about the derivative of the function at $x | 45$?

18. Find the volume of t

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20. Find the constants a and b such that the following function is continuous on the entire real number line.

$$f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ ax + b & \text{if } -1 < x < 3 \\ -2 & \text{if } x \ge 3 \end{cases}$$

a) $a = -1,\ b = -1$
b) $a = -1,\ b = 1$
c) $a = 1,\ b = -1$
d) $a = 1,\ b = 1$
e) none of the above

21. Suppose that the functions f and g are defined throughout an open interval containing the point x_0, f is differentiable at x_0, $f'(x_0) = 0$, and that g is continuous at x_0. What can be said about the differentiability of the product fg at x_0?

a) fg is differentiable at x_0 only if the function g is differentiable at x_0.
b) fg is differentiable at x_0 no matter if the function g is differentiable at x_0 or not.
c) fg is differentiable at x_0 only if the functions f and g satisfy the condition $g(x) \le f(x)$ near x_0.
d) fg is differentiable at x_0 only if $g'(x_0) = 0$.
e) none of the above

22. Find the limit: $\lim_{t \to 0} \left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} \right)$.

a) 0
b) $-\dfrac{1}{16}$
c) Does not exist
d) $-\dfrac{1}{2}$
e) none of the above

23. Find the derivative of the function $f(x) = (\sin x)^x$ for x in the interval $(0, \pi)$.

a) $(\sin x)^x\left[\ln(\sin x) + \frac{x\cos x}{\sin x}\right]$

b) $x(\sin x)^{x-1}\cos x$

c) $(\sin x)^x\left[\ln(\sin x) + \frac{x}{\sin x}\right]$

d) $\ln(\sin x) + \frac{x\cos x}{\sin x}$

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28. Find $\frac{d^{999}}{dx^{999}}\left(\sqrt{14 \sin^2 x}\right)$ for

31. Find $\int \frac{dx}{12\sin x}$.

a)

38. Find $\Big| \dfrac{dx}{\log_{\sqrt{x}} 2}$.

a) $\dfrac{x}{\log_{\sqrt{x}} 2} 4 \dfrac{x}{2\ln 2} 2\, C$

b) $2x \log_{\sqrt{x}} 2 2\, C$

c) $\dfrac{/2x \ln 2 0^{2}}{\log_{\sqrt{x}} 2} 2\; 2x \ln 2 2\, C$

d) $\dfrac{2x}{\log_{\sqrt{x}} 2} 4 \dfrac{x}{\ln 2} 2\, C$

e) none of the above

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