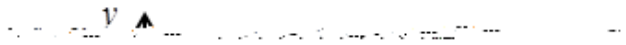
EighteenthAnnual Gainesville State College Mathematics Tournament

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

1. The following is the graph of $y = f(x)$.



Which of the following are TRUE?

I $\lim_{x \to 0} f(x) = 1$

$g(x) = (x - 1) f(x)$ is continuous at $x = 1$.

a) I

b) I, II

c) I, III

d) I, II, III

e) None of the above

2. Suppose $f(x) \quad 2$ for all x on the interval $[\ 2, 2]$. Find the value of x in $[\ 2, 2]$ at which the Mean Value Theorem is satisfied.

   a) $x \quad 0$

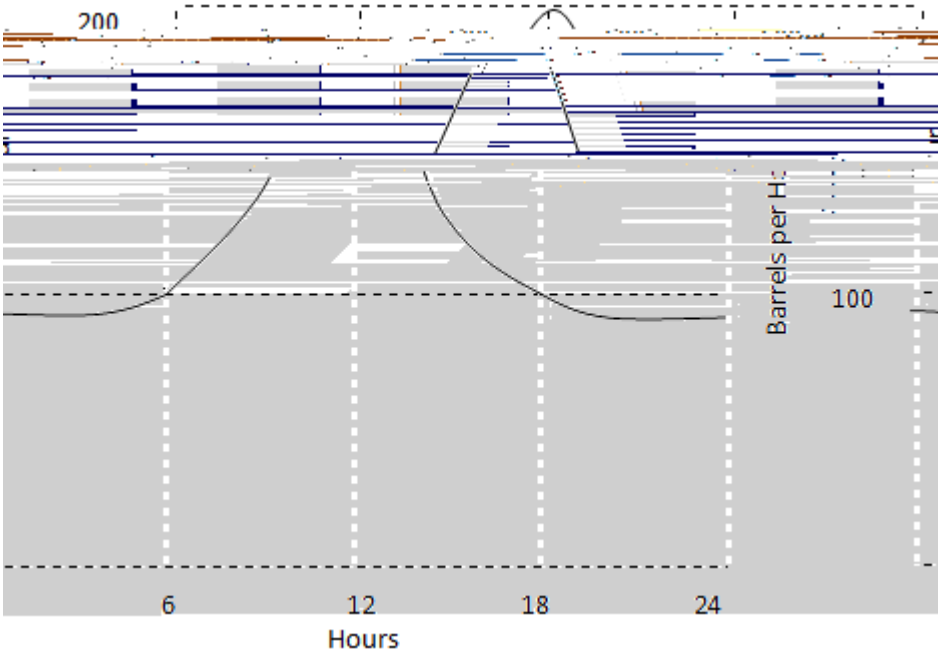
   b) $x \quad 1$

   c) $x \quad \sqrt{2}$

   d) There may be more than one value of x in $[\ 2, 2]$ at which the Mean Value Theorem is satisfied.

   e) No& 3]@h tbehbm {Ü8 \ÓØÑ& oFÓ2 bRàLete W n BT -0.016 Tc 0.016 Tw 11.8941431 627.104

5. The flow of oil (in barrels per hour) through a pipeline on April 23 is given by the graph below. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?



a) 500
b) 2400
c) 3000
d) 4800
e) None of the above

6. Define $f(1)$ in a way that extends $f(x) = \frac{x^3 - 1}{1 - x^2}$ to be continuous at $x = 1$.

a) $\frac{3}{2}$
b) $\frac{1}{2}$
c) $-\frac{1}{2}$
d) $-\frac{3}{2}$
e) None of the above

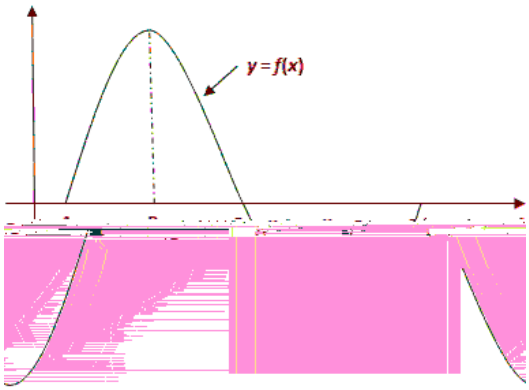
7. Suppose f is a quadratic function for which $f(0) = 1$ and $\int_{-1}^{1} f(x)\,dx = \int_{0}^{1} f(x)\,dx = \int_{1}^{0} f(x)\,dx$. Find $f(2)$.

a) 11
b) 10
c) 9
d) 8
e) None of the above

8. Find the speed v (in miles per hour) that will minimize delivery costs on a 1100-mile trip, if the cost (in dollars per hour) for fuel for the van is $C = \frac{v^2}{600}$ and the driver is paid 5 dollars per hour. (Assume there are no costs other than wages and fuel.)

a) 65.8 mi per hr
b) 55.8 mi per hr
c) 50.8 mi per hr
d) 54.8 mi per hr
e) None of the above

9. The graph of $y = f(x)$ is shown in the figure. If $g(x) = \int_{A}^{x} f(t)\,dt$, for what value of x does $g(x)$ attain its maximum?



a) A
b) B
c) C
d) D
e) None of the above

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20. If $f(x) \quad (x \quad 2)^4 (x \quad 3)^3 (x \quad 4)^2$, find $f \quad (2) \quad f \quad (3) \quad f \quad (4)$.

a) 16

b) 27

c) 0

d) 16

e) None ofthe above

21. Let $3 \quad 2\sqrt{x} \int_0^{\sqrt{x}} f(t)\,dt$. Find $f(2)$.

a) $2\sqrt{2}$

b) $3 \quad 2\sqrt{2}$

32. Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $[0, \sqrt{\ }]$ does the instantaneous rate of change of f equal the average rate of change of f on the interval?

a) Zero
b) One
c) Two
d) Three
e) None of the above

33. Let $f(x) = x^3 - 3x^2 - 1, \quad x \geq 2$. Find $f^{-1}(-1)$.

a) -1
b) 2
c) $\frac{1}{2}$
d) $\frac{1}{9}$
e) None of the above

34. Evaluate $\int_0^{/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$.

a) 0
b) $/4$
c) $/2$
d) 1
e) None of the above

35. Find the limit: $\lim_{h \to 0} \frac{\int_1^{1+h} \sqrt{x^5 + 8}\, dx}{h}$.

a) 3
b) $2\sqrt{2}$
c) 1
d) 0
e) None of the above

36. The areas A and B are bounded by the graphs of $y = e^x$, $y = xe^x$, $x = 0$, and $x = 2$, as in the picture.

Find the value of $B - A$.

a) e^{-1}
b) 2
c) e
d) $\frac{5}{2}$
e) None of the above

37. Find $\int \frac{dx}{x^{2/3} - x^{1/2}}$.

a) $2x^{1/2} - 3x^{1/3} - 6\ln\left|x^{1/6} - 1\right| + C$
b) $3x^{1/3} - 2x^{1/2} - 6\ln\left|x^{1/6} - 1\right| + C$
c) $\quad^{1/2} \quad ^{1/3} \quad \left| \quad ^{1/6} \right|$

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