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University of North Georgia

Mathematics Tournament

April 2, 2016

Solutions for the Afternoon Team Competition

Round 1

Set up the proportion $\frac{10}{3} \quad \frac{x}{321{,}000{,}000}$ where x is the area needed for the US population. Solving gives 1,070,000,000 square feet. Convert square feet to square miles giving 38.3809687787 square miles. To get the length of one side of the square, take the square root giving 6.195 miles.

Round 2

100% doesn't change the size, so we must use 80% and 150 % copies. 80% $\frac{4}{5}$ and 150% $\frac{3}{2}$

We need the smallest x and y such that $\frac{4}{5}^{x} \; \frac{3}{2}^{y} \quad \frac{324}{100}$. Simplifying gives $\frac{4^x 3^y}{5^x 2^y} \quad \frac{81}{25}$. $x \quad 2, \; y \quad 4$ works and there is no smaller combination.

Round 3

You may notice that between every time when hands are in the same position, they make 90 degrees twice. In 24 hours they will be in the same position 22 times, not counting the original time, so the answer is 44.

Round 4

The area is $A \quad R^2 \quad r^2 \quad R^2 \quad r^2$. We also have $R^2 \quad 4^2 \quad r^2$, so $R^2 \quad r^2 \quad 4^2$. So $A \quad R^2 \quad r^2 \quad 4^2 \quad 16 \quad 50.265$

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If

Round 9

Taking into account the property of the logarithm $\log_a b \quad \frac{\log_b b}{\log_b a} \quad \frac{1}{\log_b a}$, the equation can be rewritten as

Letting $c \quad \log_{18\ x\ 1^{\,2}\ x\ 7} \quad x^2 \quad 4x \quad 3$ and substituting gives $2c \quad \frac{1}{c} \quad 3$ and then the quadratic equation $2c^2 \quad 3c \quad 1 \quad 0$. Solving the quadratic gives the 2 solutions $c \quad 1$ and $c \quad \frac{1}{2}$.

Now we have 2 cases. Case 1: $c \quad 1$ Recalling that $c \quad \log_{18\ x\ 1^{\,2}\ x\ 7} \quad x^2 \quad 4x \quad 3$, we have

. Solving this logarithmic equation, we confirm that there are no integer solutions for this case. Case 2: $c \quad \frac{1}{2}$ We have $\log_{18\ x\ 1^{\,2}\ x\ 7} \quad x^2 \quad 4x \quad 3 \quad \frac{1}{2}$. We can rewrite as . Factoring gives $x \quad 3 \quad x \quad 1 \quad \sqrt{18} \quad x \quad 1\sqrt{x \quad 7}$, but $x \quad 1$, so $x \quad 1$ is not a solution. Now we have $x \quad 3 \quad \sqrt{18}\sqrt{x \quad 7}$. Squaring both sides gives $x^2 \quad 6x \quad 9 \quad 18 \quad x \quad 7$ and then $x^2 \quad 24x \quad 135 \quad 0$ which gives the two integer solutions $x \quad 15$ and $x \quad 9$.

Round 10

Let $5 \quad \cfrac{6}{5 \quad \cfrac{6}{\vdots}} \quad x$, then $5 \quad \frac{6}{x} \quad x$ and $x^2 \quad 5x \quad 6 \quad 0$. Factoring gives $x \quad 6 \quad x \quad 1 \quad 0$. This gives solutions $x \quad 6$ and $x \quad 1$, but $x \quad 1$. Similarly, let $16 \quad \cfrac{64}{16 \quad \cfrac{64}{\vdots}} \quad x$, then $16 \quad \frac{64}{x} \quad x$ and $x^2 \quad 16x \quad 64 \quad 0$. Factoring gives $x \quad 8 \quad x \quad 8 \quad 0$. This gives the solution $x \quad 8$. Thus,

$$\cfrac{6}{5 \quad \cfrac{6}{5 \quad \cfrac{6}{\vdots}}} \quad \cfrac{8}{16 \quad \cfrac{64}{16 \quad \cfrac{64}{\vdots}}} \quad \frac{6}{6} \quad \frac{8}{8} \quad 2.$$